Geometry of strings and branes

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Chapter 3

The DW/QFT correspondence

It has been known for some time that the geometry of a large class of p-branes interpolates between the near-horizon geometry $AdS_{p+2} \times S^{D-p-2}$ and the asymptotic geometry $\mathbb{R}^{1,D-1}$ [82]. This interpolating structure becomes apparent when one studies the geometry in the sigma-model frame of the magnetically dual brane – the so-called dual frame [83].

Soon after the discovery of the AdS/CFT correspondence, the connection between the geometry and worldvolume theory was therefore also investigated for other Dp-branes [111]. In contrast to the D3-brane, Dp-branes generically have a non-vanishing dilaton; this breaks the conformal invariance possessed by the AdS near-horizon geometry. The worldvolume theory of generic Dp-branes is also not a conformal field theory, but rather a more general quantum field theory (QFT).

It was shown in [112] that p-brane solutions having an AdS_{p+2} near-horizon geometry and a non-vanishing dilaton fall in a specific class of domain-walls, which we will denote by DW_{p+2}. Anti-de-Sitter spacetime then becomes the special case that the dilaton vanishes. Domain-wall spacetimes naturally occur in massive supergravities; theories with a mass parameter or a cosmological constant [113].

The developments above inspired the authors of [114] to conjecture a DW/QFT correspondence for ten-dimensional Dp-branes. They conjectured that gravity on a domain-wall spacetime should be holographically dual to a quantum field theory on a slice of that spacetime. We have generalized this correspondence for general p-branes in arbitrary dimensions [15]. The mapping between classical supergravity and a strongly coupled field theory persists for more general Dp-branes. However, the lack of conformal invariance forms an obstruction for making quantitative checks on the DW/QFT correspondence in this case.

Instead of considering the holographic duals of more general brane solutions, one can also study the gravity duals of more general quantum field theories. In particular, relevant deformations of conformal field theories turn out to be of interest. Such deformations induce a renormalization group (RG) flow in the space of coupling constants. The gravity duals of

RG-flows between fixed points of the field theory beta-function, can be seen as domain-walls interpolating between Anti-de-Sitter vacua of the scalar potential of a supergravity theory.

We will start this chapter with describing the near-horizon geometry of a class of two-block p-brane solutions. After that, we will present a class of domain-wall solutions that contain Anti-de-Sitter spacetime as a special case. We will indicate how a sphere reduction of the supergravity action supporting the original p-brane gives rise to a domain-wall solution of a gauged supergravity. These results have been published in [15], and an abridged version appeared as a proceedings in [115].

We will finish this chapter with indicating how RG-flows of conformal field theories are related to supergravity solutions that interpolate between different Anti-de-Sitter vacua; this will provide the transition to chapter 4, where we will describe brane world scenarios.

3.1 Near-horizon geometries of p-branes

In this section we will look in more detail into the geometrical properties of the class of two-block p-brane solutions of section 1.4.1.

3.1.1 Two-block solutions

Our starting point is the magnetic formulation of the generic supergravity action in the Einstein frame in (1.57)

$$\mathcal{L}^{\mathrm{E}}_{(D,\tilde{p})} = R \star \mathbb{1} - \frac{4}{D-2} \star \mathrm{d}\phi \wedge \mathrm{d}\phi - \frac{1}{2} e^{-a\phi} g_s^{2-4k} \star F_{(\tilde{p}+2)} \wedge F_{(\tilde{p}+2)} \,. \tag{3.1}$$

We recall that this action supports an electric p-brane solution given by (1.53)

$$\text{electric } p\text{-brane} = \begin{cases} \mathrm{d}s^2_{\mathrm{E}} &= H^{\frac{-4\tilde{d}}{(D-2)\Delta}} \mathrm{d}x^2_{(d)} + H^{\frac{4d}{(D-2)\Delta}} \mathrm{d}y^2_{(\tilde{d}+2)} \,, \\ e^{\Phi} &= g_s H^{\frac{(D-2)a}{4\Delta}} \,, \\ F_{(p+2)} &= g_s^{-k} \sqrt{\frac{4}{\Delta}}\, \mathrm{d}^d x \wedge \mathrm{d}H^{-1} \,, \\ H(y) &= 1 + \left(\frac{R}{y}\right)^{\tilde{d}} \,. \end{cases} \tag{3.2}$$

The parameter Δ is defined as

$$\Delta = \frac{(D-2)a^2}{8} + \frac{2d\tilde{d}}{(D-2)} \,. \tag{3.3}$$

We also recall that we restrict the worldvolume dimension of the dual brane to be strictly positive. The case $\tilde{d} = 0$ corresponds to $(D-3)$-branes, which have a logarithmic harmonic function. The case $\tilde{d} = -2$ corresponds to spacetime filling branes. We will not consider such branes in our subsequent analysis. The branes with $\tilde{d} = -1$ are $(D-2)$-branes; their asymptotic geometries are not given by flat spacetime. They can be viewed as domain-walls, and we will discuss them in the next section.

For the D3-brane, the Einstein frame coincides with both the sigma-model frame and the dual frame, but for more general branes this is not the case. The dual frame is the most useful for our purposes. Recall that after the rescaling (1.84)

$$g^{\mathrm{D}}_{\mu\nu} = e^{\omega_{\mathrm{D}}\phi} g^{\mathrm{E}}_{\mu\nu}\,, \quad \omega_{\mathrm{D}} = \frac{a}{\tilde{d}}\,, \tag{3.4}$$

the action will simplify to the form (1.85)

$$\mathcal{L}^{\mathrm{D}}_{(D,\,\tilde{p})} = e^{\delta_{\mathrm{D}}\phi}\left(R \star \mathbb{1} + \gamma_{\mathrm{D}} \star \mathrm{d}\phi \wedge \mathrm{d}\phi - \frac{1}{2} \star F_{(\tilde{p}+2)} \wedge F_{(\tilde{p}+2)}\right)\,. \tag{3.5}$$

The overall dilaton factor and the modified kinetic term are given by (1.86)

$$\delta_{\mathrm{D}} = -\frac{(D-2)a}{2\tilde{d}}\,, \quad \gamma_{\mathrm{D}} = \frac{D-1}{D-2}\delta^2_{\mathrm{D}} - \frac{4}{D-2}\,. \tag{3.6}$$

In this dual frame, the metric is given by

$$\mathrm{d}s^2_{\mathrm{D}} = H^{\left(\frac{2}{\tilde{d}} - \frac{4}{\Delta}\right)} \mathrm{d}x^2_{(d)} + H^{\frac{2}{\tilde{d}}}\left(\mathrm{d}y^2 + y^2 \mathrm{d}\Omega^2_{(\tilde{d}+1)}\right)\,. \tag{3.7}$$

3.1.2 The near-horizon limit

If we now take the near-horizon limit

$$\frac{y}{R} \to 0\,, \tag{3.8}$$

then we find for the near-horizon geometry and dilaton dependence of the electric p-brane in the dual frame

$$\mathrm{d}s^2_{\mathrm{D}} = \left(\frac{R}{y}\right)^{\left(2 - \frac{4\tilde{d}}{\Delta}\right)} \mathrm{d}x^2_{(d)} + \left(\frac{R}{y}\right)^2 \mathrm{d}y^2 + R^2 \mathrm{d}\Omega^2_{(\tilde{d}+1)}\,, \quad e^{\Phi(y)} = g_s \left(\frac{R}{y}\right)^{\frac{(D-2)\tilde{d}a}{4\Delta}}\,. \tag{3.9}$$

This looks similar to Anti-de-Sitter spacetime in Poincaré coordinates (2.31). Specifically, we take

$$e^{-r/R} = \frac{y}{R}\,. \tag{3.10}$$

In these coordinates, the metric and the dilaton take on the form

$$\mathrm{d}s^2_{\mathrm{D}} = e^{(2 - \frac{4\tilde{d}}{\Delta})r/R} \mathrm{d}x^2_{(d)} + \mathrm{d}r^2 + R^2 \mathrm{d}\Omega^2_{(\tilde{d}+1)}\,, \quad \phi(r) = \frac{(D-2)\tilde{d}ar}{4\Delta R}\,. \tag{3.11}$$

The analog of the horospherical coordinates (2.29) is given by

$$\frac{U}{L} = e^{-r/L}\,, \quad L = \frac{R}{\left(\frac{2\tilde{d}}{\Delta} - 1\right)}\,. \tag{3.12}$$

In these coordinates, the metric takes on the form

$$\begin{aligned} ds^2_{\mathrm{D}} &= \left(\frac{U}{L}\right)^2 dx^2_{(d+1)} + \left(\frac{L}{U}\right)^2 dU^2 + R^2 d\Omega^2_{(\tilde{d}+1)} \\ &\equiv AdS_{d+1}\,(L) \times S^{\tilde{d}+1}(R)\,. \end{aligned} \tag{3.13}$$

3.1.3 Interpolating solitons

From this, we deduce that the near-horizon geometry for the two-block p-branes is given by $AdS_{p+2} \times S^{D-p-3}$ in the background of a dilaton depending linearly on the radial AdS-coordinate. Anticipating the discussion on domain-walls in the following section, we will call such geometries $DW_{p+2} \times S^{D-p-3}$. The size of the Anti-de-Sitter is proportional to the size of the sphere, as can be seen from (3.12).

There are two special case to be considered: $a = 0$, and $\tilde{d} = \frac{\Delta}{2}$. The first case corresponds to branes having no dilaton. Examples of this case are the D3-brane in ten dimensions and the eleven-dimensional M2-brane and M5-brane. These branes have a pure Anti-de-Sitter spacetime in their near-horizon geometry. In [15], a table of all cases with $a = 0$ was given.

The second case corresponds to branes with an infinite AdS-radius. For such a radius, the cosmological constant (2.44) vanishes, and the spacetime becomes conformally flat. The near-horizon becomes $\mathbb{R}^{1,p+1} \times S^{D-p-3}$. Examples of such spaces are five-branes in ten-dimensions, which have $\mathbb{R}^{1,6} \times S^3$ as their near-horizon geometry. For more details and examples, we refer to [15].

On the other hand, taking

$$\frac{y}{R} \to \infty\,, \tag{3.14}$$

the harmonic function becomes constant, and the metric describes Minkowski space $\mathbb{R}^{1,D-1}$. This means that we can view a p-brane as a gravitational soliton with a geometry that interpolates between the near-horizon geometry $DW_{p+2} \times S^{D-p-3}$ and the asymptotic geometry $\mathbb{R}^{1,D-1}$.

3.2 Domain-walls

Domain-walls can be defined as topological defects of co-dimension one. They separate a spacetime (or a phase space) into several domains along a single coordinate. In the presence of domain-walls, physical parameters can generically taken to be piecewise smooth. However, on an intersection of two domains – the domain-wall – such a parameter is generically not differentiable or even continuous, and its derivatives can have delta-function singularities. These properties make domain-walls useful for describing physical processes such as phase transitions.

In this section, we will describe a class of such domain-walls that occur in supergravity theories. We will make a distinction between "thin" domain-walls and "thick" domain-walls. The former class can be viewed as a single $(D-2)$-brane placed in the origin of the y-coordinate, separating the spacetime into two regions. In each region, a characteristic magnetic field-strength can be defined that changes its sign across the brane. At $y=0$, there is a curvature singularity.

In section 3.3.3, we will describe "thick" domain-walls; they can be viewed as smoothly interpolating solitons between different supergravity vacua, without having singularities. They have no direct brane-interpretation, but they can sometimes be related to intersecting branes in a higher-dimensional spacetime.

3.2.1 Solution Ansatz

We will now discuss domain-walls which support only a single scalar and a d-dimensional gauge potential. The action in the Einstein frame is

$$\mathcal{L}^{\mathrm{E}}_{\mathrm{domain}} = R \star \mathbb{1} - \frac{4}{d-1} \star \mathrm{d}\varphi \wedge \mathrm{d}\varphi - \frac{1}{2} e^{b\varphi} g_s^{2\bar{k}} \star F_{(d+1)} \wedge F_{(d+1)} \,, \tag{3.15}$$

where the dilaton exponential factor is obtained from (1.88)

$$\bar{k} = \frac{b}{2} + \frac{2d}{d-1} \,. \tag{3.16}$$

The domain-wall solution is analogous to the general electric p-brane

$$\text{domain-wall} = \begin{cases} \mathrm{d}s^2_{\mathrm{E}} &= H^{\frac{-4\varepsilon}{(d-1)\Delta_{\mathrm{dw}}}} \mathrm{d}x^2_{(d)} + H^{\frac{-4d\varepsilon}{(d-1)\Delta_{\mathrm{dw}}} - 2(\varepsilon+1)} \mathrm{d}y^2 \,, \\ e^{\varphi} &= H^{\frac{-(d-1)b\epsilon}{4\Delta_{\mathrm{dw}}}} \,, \\ F_{(d+1)} &= g_s^{-\bar{k}} \sqrt{\frac{4}{\Delta_{\mathrm{dw}}}}\, \mathrm{d}^d x \wedge \mathrm{d}H^{\varepsilon} \,, \\ H(y) &= 1 + Q|y| \,. \end{cases} \tag{3.17}$$

with the parameter Δ_{dw} given by

$$\Delta_{\mathrm{dw}} = \frac{(d-1)b^2}{8} - \frac{2d}{d-1} \,. \tag{3.18}$$

If the dilaton vanishes, the parameter Δ_{dw} reduces to

$$\Delta_{\mathrm{AdS}} = -\frac{2d}{d-1} \,. \tag{3.19}$$

This is a lower-bound on Δ_{dw}, and we can classify the domain-wall solutions into four classes, depending on whether

1. $\Delta_{\rm dw} = \Delta_{\rm AdS}$,
2. $\Delta_{\rm AdS} < \Delta_{\rm dw} < 0$,
3. $\Delta_{\rm dw} = 0$,
4. $\Delta_{\rm dw} > 0$.

We will not consider the third category. It will turn out that domain-walls which are reductions of branes in higher dimensions fall into categories 1 or 2. Elementary domain-walls fall into category 4.

The domain-wall (3.17) is a one-parameter class of solutions: the parameter ε cannot be determined, in contrast with normal p-branes which have $\varepsilon = -1$. Even though there exists no magnetically dual for the domain-wall, we can still define a magnetically dual field-strength

$$F_{(0)} \equiv e^{b\varphi} g_s^{\bar{k}} \star F_{(d+1)} \,. \tag{3.20}$$

We can eliminate ε if we define a mass parameter as $m = Q\varepsilon$. Using the form of $F_{(d+1)}$ given in (3.17), we see that the magnetic field-strength changes its sign across the point $y = 0$; this is the position where the brane is located.

It is straightforward to check that the invariant volume form is given by

$$e^{2b\varphi} H^{2(\varepsilon-1)} \mathrm{d}^d x \wedge \mathrm{d}y = \star \mathbb{1} \,. \tag{3.21}$$

Using this, we can express the action in the magnetic formulation as

$$\mathcal{L}^{\rm E}_{\rm domain} = R \star \mathbb{1} - \frac{4}{d-1} \star \mathrm{d}\varphi \wedge \mathrm{d}\varphi - 2e^{-b\varphi} \Lambda \star \mathbb{1} \,. \tag{3.22}$$

The cosmological constant is given by

$$\Lambda = \frac{m^2}{\Delta_{\rm dw}} \,. \tag{3.23}$$

3.2.2 Asymptotic geometry

Even though domain-walls do not have a magnetically dual brane, it is again it useful to transform to the dual frame. This frame is defined by (1.84), with $\tilde{d} = -1$

$$g^{\rm D}_{\mu\nu} = e^{-b\varphi} g^{\rm E}_{\mu\nu} \,. \tag{3.24}$$

The action (3.22) now has an overall dilaton factor and a modified kinetic term

$$\mathcal{L}^{\rm D}_{\rm domain} = e^{\bar{\delta}_{\rm D}\varphi} \left(R \star \mathbb{1} + \bar{\gamma}_{\rm D} \star \mathrm{d}\varphi \wedge \mathrm{d}\varphi - 2\Lambda \right) \,, \tag{3.25}$$

with $\bar{\delta}_{\rm D}$ and $\bar{\gamma}_{\rm D}$ given by

$$\bar{\delta}_{\rm D} = \frac{(d-1)b}{2}\,, \quad \bar{\gamma}_{\rm D} = \frac{d}{d-1}\bar{\delta}_{\rm D}^2 - \frac{4}{d-1} \tag{3.26}$$

The metric in the dual frame reads

$$\mathrm{d}s_{\rm D}^2 = H^{\frac{2(\Delta_{\rm dw}+2)\varepsilon}{\Delta_{\rm dw}}}\mathrm{d}x_{(d)}^2 + H^{-2}\mathrm{d}y^2\,. \tag{3.27}$$

In the previous section, we have shown that generic p-branes could be interpreted as solutions which interpolate between two different supergravity vacua: Minkowski spacetime $\mathbb{R}^{1,D-1}$ asymptotically away from the brane and $AdS_{d+1}\times S^{\tilde{d}+1}$ near the brane. For domain-walls, this is not the case: they are not asymptotically flat. To discover what asymptotic geometry they have, we take the limit

$$Q|y| \to \infty\,. \tag{3.28}$$

The metric and the dilaton then take the form

$$\mathrm{d}s_{\rm D}^2 = (Qy)^{\frac{2(\Delta_{\rm dw}+2)\varepsilon}{\Delta_{\rm dw}}}\mathrm{d}x_{(d)}^2 + (Qy)^{-2}\mathrm{d}y^2\,, \quad e^{\varphi(y)} = (Qy)^{\frac{-(d-1)b\epsilon}{4\Delta_{\rm dw}}}\,. \tag{3.29}$$

We can now exponentiate the y-coordinate

$$e^{-Qr} = Qy\,, \tag{3.30}$$

after which the metric has the form of Anti-de-Sitter spacetime in Poincaré coordinates, and the dilaton now has a linear dependence on the radial AdS-coordinate

$$\mathrm{d}s_{\rm D}^2 = e^{-\frac{2(\Delta_{\rm dw}+2)}{\Delta_{\rm dw}}mr}\mathrm{d}x_{(d)}^2 + \mathrm{d}r^2\,, \quad \varphi(r) = \frac{(d-1)mb}{4\Delta_{\rm dw}}r\,. \tag{3.31}$$

After going to horospherical coordinates

$$\frac{U}{L} = e^{-r/L}\,, \quad L = \frac{\Delta_{\rm dw}}{(\Delta_{\rm dw}+2)m}\,, \tag{3.32}$$

we get for the metric

$$\begin{aligned} \mathrm{d}s_{\rm D}^2 &= \left(\frac{U}{L}\right)^2\mathrm{d}x_{(d+1)}^2 + \left(\frac{L}{U}\right)^2\mathrm{d}U^2 \\ &\equiv AdS_{d+1}\,(L)\,. \end{aligned} \tag{3.33}$$

From the above, we see that $(D-2)$-branes are different from generic p-branes. They do not interpolate between flat spacetime and a product of Anti-de-Sitter spacetime times a sphere. Instead, they form an interpolation between two asymptotic Anti-de-Sitter spacetimes, with a dilaton depending linearly on the radial AdS-coordinate.

3.2.3 Sphere reductions

A p-brane in $(p+2)$ dimensions can be seen as a domain-wall. Hence, if we reduce the action (1.85) over the sphere $S^{\tilde{d}+1}$, we expect to find a domain-wall described by the action

$$\mathcal{L}^{\mathrm{D}}_{(d+1,\,p)} = e^{\delta_{\mathrm{D}}\phi}\left(R \star \mathbb{1} + \gamma_{\mathrm{D}} \star \mathrm{d}\phi \wedge \mathrm{d}\phi - 2\Lambda \star \mathbb{1}\right) . \tag{3.34}$$

Up to a dilaton rescaling, this is of the same form as (3.25). We can determine the scale factor from[1]

$$\phi = c\varphi \rightarrow c^2 = \frac{\bar{\gamma}_{\mathrm{D}}}{\gamma_{\mathrm{D}}} = \left(\frac{\bar{\delta}_{\mathrm{D}}}{\delta_{\mathrm{D}}}\right)^2 . \tag{3.35}$$

Combining (1.86) and (3.26) with either (1.54) or with (3.18), we can express the dilaton rescaling in two ways

$$c^2 = \frac{2\tilde{d}^2}{\Delta + (\Delta - 2)\tilde{d}} = -\frac{\Delta_{\mathrm{dw}} + (\Delta_{\mathrm{dw}} + 2)\tilde{d}}{2} . \tag{3.36}$$

This means that we can express the parameter Δ_{dw} of the $(d+1)$-dimensional domain-wall in terms of the parameter Δ of the original D-dimensional p-brane solution

$$\Delta_{\mathrm{dw}} = \frac{-2\tilde{d}\Delta}{\Delta + (\Delta - 2)\tilde{d}} . \tag{3.37}$$

The dilaton couplings a and b in the Einstein frame actions (1.50) and (3.15) are then related by

$$b = -a\frac{c(D-2)}{\tilde{d}(d-1)} . \tag{3.38}$$

Furthermore, comparing the sizes of the Anti-de-Sitter spacetime given in (3.12) and (3.32), we deduce

$$m = \frac{\tilde{d}}{R} . \tag{3.39}$$

Finally, we can also relate the cosmological constant of the reduced brane solution in terms of parameters of the original brane solution

$$\Lambda = -\frac{\tilde{d}}{2R^2}\left((\tilde{d}+1) - \frac{2\tilde{d}}{\Delta}\right) . \tag{3.40}$$

So far, we have shown that the near-horizon geometry of a generic p-brane is given by $AdS_{p+2} \times S^{D-p-3}$, and that the reduction of this geometry over the sphere can be related

[1] This is a refinement w.r.t. [15] where the same scale factor was obtained by transforming the reduced action (3.34) back to the Einstein frame, and comparing this with (3.15). However, such a rescaling is singular for $d = 0$. We avoid this slight complication by the method sketched above.

to a domain-wall in $(p+2)$-dimensions. Since the action (1.50) is a consistent truncation of a more general supergravity action, this would suggest that a sphere-reduction of this more general supergravity action leads to a lower-dimensional supergravity action, of which (3.15) should be a consistent truncation.

In section 2.3.2, we saw that the AdS/CFT formulation was most conveniently formulated as a duality between $\mathrm{SO}(6)$ gauged $\mathcal{N}=8$ supergravity in $D=5$, and $\mathcal{N}=4$ Yang-Mills theory in $D=4$. The former theory is conjectured to be a consistent truncation of the S^5-reduction of Type IIB supergravity in $D=10$. A natural form of the DW/QFT correspondence would then be in terms of a duality between an $\mathrm{SO}(\tilde{d}+2)$ gauged supergravity in $d+1$ dimensions, and the worldvolume theory of the corresponding p-brane in d dimensions [114].

The underlying assumption of such a scheme is that it is possible to consistently truncate the $S^{\tilde{d}+1}$-reduction of the higher-dimensional supergravity to only the massless Kaluza-Klein modes. The Anti-de-Sitter spacetime and the sphere are of comparable radius, as we showed in (3.12). This is fundamentally different from, say, Calabi-Yau compactifications of string theory, where the consistency is at least approximately guaranteed by taking the compactification radius to zero, thereby automatically decoupling all the higher Kaluza-Klein modes. Consistent truncations of sphere reductions are in general hard to find. Until recently, only the gauged maximally supersymmetric supergravities in $D=4$ [116] and $D=7$ [117] were shown to be consistent truncations of the compactifications of eleven-dimensional supergravity on S^7 [118] and S^4 [119].

For sphere reductions, if one wants to keep the massless gauge fields generating the $\mathrm{SO}(\tilde{d}+2)$ symmetry, one generically also needs to keep most, if not all, scalar fields coming from the reduction Ansatz of the metric and the $(p+1)$-form gauge potential. This complicated matters enormously: e.g. the S^5 reduction of Type IIB supergravity results in 42 scalars in $D=5$ which interact in a non-linear fashion. Moreover, the Killing vectors on the sphere need to satisfy certain consistency conditions. These conditions turn out to be hard to satisfy, precisely only for the examples mentioned above does a maximally supersymmetric gauged supergravity form a consistent truncation of a sphere-compactification [120].

Nevertheless, many new results on consistent sphere reductions have been obtained in recent years following the AdS/CFT correspondence. In particular, it is possible to consider truncations of the complete massless Kaluza-Klein sector to only the subset of the lower-dimensional scalars that transform in the Cartan-subalgebra of the gauge group. These truncated gauged supergravities have solutions that can be lifted to solutions of the original supergravity theory. There, they correspond to an infinite stack of overlapping branes [121].

3.3 Quantum field theory

In this section, we will explore what field theory information can be extracted from the DW/QFT correspondence. In particular for the class of Dp-branes and intersections thereof, we will derive the scaling dependence of the corresponding worldvolume theory coupling constants. The end of this section will review how the renormalization-group flow induced by relevant deformations of conformal theories give rise to domain-wall solutions that interpolate between different supergravity vacua.

3.3.1 Dual worldvolume theories

The geometrical structure of a large class of p-branes in the dual frame is rather similar. However, the worldvolume theories of these branes are much more diverse, as we saw in section 1.4.2. We will parameterize the worldvolume action to first approximation as a theory described by a q-form gauge field. The cases $q=0$, $q=1$, and $q=2$ then correspond to a an action describing a scalar, a vector, and a tensor multiplet, respectively.

$$S_{\rm brane} = -\tau_p \int \mathrm{d}^{p+1}\sigma \, (\ell_s^{q+1} F_{(q+1)})^2 + \dots \tag{3.41}$$

$$\equiv -\frac{1}{g_{\rm gauge}^2} \int \mathrm{d}^{p+1}\sigma \, F_{(q+1)}^2 \, . \tag{3.42}$$

The mass-dimension of the field-theory coupling constant can be obtained from the general expression for a p-brane tension

$$g_{\rm gauge}^2 = g_s^k \ell_s^\alpha \, , \quad \alpha = p - 2q - 1 \, . \tag{3.43}$$

At a given energy scale E, a dimensionless coupling constant is defined as

$$\lambda(E) \equiv g_{\rm gauge}^2 E^\alpha \, . \tag{3.44}$$

In the case of the D3-brane, we saw that there were two natural energy scales: the energy $E_{\rm W}$ of open strings stretching between the stack of N D3-branes and a single D3-brane probe, and the energy E_ψ of a supergravity field ψ probing the N D3-branes.

For general p-branes, the holographic energy scale can be obtained from an analysis of the wave equation for a supergravity scalar field ψ. The analog of (2.18) for a general p-brane is

$$E_\psi \equiv u = \frac{y^\beta}{R^{\beta+1}} \, , \quad \beta = \frac{2\tilde{d}}{\Delta} - 1 \, . \tag{3.45}$$

Of all the brane solutions in string theory, Dp-branes have been studied most. In particular, they have a exact description in conformal field theory as boundary states [122]. Other branes, such as the NS5-brane, are also believed to form coherent states in the conformal field theory description of string theory, but a precise understanding is lacking. This suggests that we should consider p-branes which are closely related to Dp-branes.

If we also consider Dp-brane probes of the p-brane, then we have an additional energy scale equivalent to (2.16)

$$E_W \equiv U = \frac{y}{\ell_s^2} \, . \tag{3.46}$$

For general p-branes, it is not possible to choose a Dp-brane as a sensible probe. The reason is that we would like the dimensionless coupling constants of both energy scales to be related independently of the near-horizon limit. In particular, looking at the y-dependence of the dimensionless coupling constants constructed from the two energy scales we expect that

$$\lambda(u) = \lambda(U)^{\beta} \,. \tag{3.47}$$

For this to happen, the g_s and N dependence on both sides will also have to match. This gives two restrictions

$$k = 1 \,, \quad \alpha = \Delta - \tilde{d} \,. \tag{3.48}$$

The first constraint has an obvious interpretation; it says that the dilaton dependence of the p-brane tension is the same as for a Dp-brane, namely $\tau_p = \frac{1}{g_s}$. The second constraint is more surprising, it gives the mass dimension of the coupling constant on the p-brane worldvolume. If we combine the constraint (3.48) with the expression for Δ (1.54), then we find

$$a = \frac{2(D - 2(2+p))}{D-2} \,, \quad \Delta = \frac{D-2}{2} \,, \quad \alpha = -a\frac{D-2}{4} \,. \tag{3.49}$$

This means that the scale dependence of the worldvolume coupling constant is proportional to the dilaton dependence of the gauge field kinetic term in the action. In particular, p-branes that do not couple to the dilaton have a scale-independent coupling constant in their worldvolume theory.

The supergravity approximation is valid as long as the string tension in the dual frame is large. We can calculate this with the same scaling arguments as we used in deriving the effective brane tension in the string frame

$$\tau_s^{\mathrm{D}} = \frac{\lambda(U)^{\frac{2}{\Delta}}}{\ell_s^2} \,. \tag{3.50}$$

Quantum corrections in string theory are controlled through the dilaton which is now not a constant g_s, as in (2.23), but is instead given by

$$e^{\Phi} = \frac{\lambda(U)^{\frac{\tilde{d}}{\Delta}}}{N} \,. \tag{3.51}$$

The ratio of the two different energy scales can be expressed in a similar form as (2.24)

$$\frac{U}{u} = \lambda(U)^{\frac{2}{\Delta}} \,. \tag{3.52}$$

The generalization of table 2.1 is given in table 3.1. It gives the relations between the ranges of the various parameters on both sides of the theory for which one side becomes computationally feasible.

In the remainder of this section, we will discuss some specific examples of worldvolume theories of the p-branes discussed in this chapter. For more details see [15].

Regime	Gravity	Gauge theory
Perturbative field theory	$\tau_s^{\mathrm{D}} \ell_s^2 \ll 1$	$\lambda(U) \ll 1$
Classical string theory	$e^{\Phi} \ll 1$	$\frac{\lambda(U)^{\frac{\tilde{d}}{\Delta}}}{N} \ll 1$
Supergravity	$\tau_s^{\mathrm{D}} \ell_s^2 \gg 1$	$\lambda(U)^{\frac{2}{\Delta}} \gg 1$

Table 3.1: Regimes of the DW/QFT correspondence.

Ten-dimensional Dp-branes

The first class of examples is formed by the ten-dimensional Dp-branes. They have $a = \frac{3-p}{2}$ and $\Delta = 4$, in accordance with (3.49), from which we also deduce that $\alpha = p-3$. Comparing this with (3.43), we deduce that $q = 1$. In other words, the coupling constant on the Dp-brane worldvolume scales consistently with the vector multiplet description of the worldvolume theory.

The regime where perturbative field theory is possible is when $\lambda(U) \ll 1$. The sign of α is positive for $p > 3$ and negative for $p < 3$. This means that, for the Dp-branes with $p < 3$, the perturbative field theory description is valid for large U – the gauge theory is UV-free. The field theories of Dp-branes with $p > 3$ can be treated perturbatively for small U – the IR regime.

Since the conformal symmetry of the D3-brane worldvolume theory does not extend to the Dp-brane worldvolume theories, there are hardly any quantitative tests available. However, the qualitative structure of the phase diagram of these theories as a function of N, U and λ has been investigated in [111], and the relation to gauged supergravities has been studied in [114].

Six-dimensional dp-branes

Intersections of a Dp-brane with a D$(p+4)$-brane in which the smaller brane lies entirely inside the larger brane, are denoted as $(p|\mathrm{D}p, \mathrm{D}(p+4))$. These intersections preserve half the supersymmetries of the constituent D-branes and this means that they have $\Delta = 2$.

Generically, a Dp-brane has an $\mathcal{N} = 4$ vector-multiplet on its worldvolume. In the presence of D$(p+4)$-branes, one can split these degrees of freedom into an $\mathcal{N} = 2$ vector multiplet parallel to the D$(p+4)$-brane and an $\mathcal{N} = 2$ hypermultiplet transverse to both D-branes [123]. The strings stretching between branes of different dimension have the interpretation of quarks on the worldvolume of the Dp-branes, whereas the strings starting and ending on Dp-branes have the usual interpretation of gauge fields.

If there are N_{c} Dp-branes and N_{f} D$(p+4)$-branes, then $\mathrm{U}(N_{\mathrm{c}})$ acts as the color group and $\mathrm{U}(N_{\mathrm{f}})$ as the flavor group [123]. After a dimensional reduction of the four transverse coordinates of both branes, they form a stack of $N = N_{\mathrm{c}} + N_{\mathrm{f}}$ six-dimensional p-branes called dp-branes [15]. Comparing with (3.49), we see that such branes indeed have $\Delta = 2$ as well as $a = 1 - p$ and $\alpha = p - 1$. From (3.43), we deduce that $q = 0$ for such branes; their worldvolume theory should consist of a hypermultiplet.

This result is not too surprising. First of all, the vector multiplet corresponding to the fluctuations parallel to the D$(p+4)$-brane is lost in the dimensional reduction process. Moreover, scalar multiplets have spins in a range which is twice as small as that of vector multiplets. This is consistent with the ratio of the amounts of supersymmetries preserved by dp-branes and Dp-branes.

For $p = 1$, the worldvolume is a two-dimensional conformal field theory and the near-horizon geometry is $AdS_3 \times S^3$ without a dilaton background. This is the most studied example [124]; it corresponds to the $(1|\mathrm{D}1, \mathrm{D}5)$ system in ten dimensions compactified on a four-dimensional torus [125]. In this case, there is also some progress in the area of treating string theory on the curved AdS_3 spacetime [126].

3.3.2 Deformations and renormalization

Up to now, we have discussed the most obvious deformation of the D3-brane system: Dp-branes, and intersections thereof. However, the worldvolume theories of these branes do not lend themselves for a computationally feasible extension of the AdS/CFT correspondence, as we have seen in the previous section.

Another way of generalizing the AdS/CFT correspondence is to look at deformations of the conformal field theory that is a dual description of gravity around an AdS spacetime. In general, such deformations will break the conformal invariance and not much information can be obtained. However, as will be made precise below, for so-called relevant deformations, the theory can flow to another conformal theory.

The AdS/CFT correspondence provides the field theory with two natural energy scales: the Dp-brane probe energy U, and the holographic energy u. The formalism which deals most efficiently with field theories having an energy scale is called effective field theory. For a good review, we refer to [127].

In a field theory with an energy scale Λ, one can make a distinction between the momentum modes of a field into high-frequency and low-frequency modes

$$\{\phi(\omega)\} = \{\phi(\omega)_{\mathrm{L}}\} + \{\phi(\omega)_{\mathrm{H}}\} . \tag{3.53}$$

The obvious definitions are given by

$$\begin{aligned} \{\phi(\omega)_{\mathrm{L}}\} &= \{\phi(\omega) : \omega < \Lambda\} , \\ \{\phi(\omega)_{\mathrm{H}}\} &= \{\phi(\omega) : \omega > \Lambda\} . \end{aligned} \tag{3.54}$$

An effective field theory is obtained by integrating out the high-frequency modes in the (Euclidean) path integral

$$\int \mathcal{D}\phi_{\mathrm{L}} \int \mathcal{D}\phi_{\mathrm{H}} e^{-S(\phi_{\mathrm{L}},\phi_{\mathrm{H}})} \equiv \int \mathcal{D}\phi_{\mathrm{L}} e^{-S_\Lambda(\phi_{\mathrm{L}})} . \tag{3.55}$$

This defines the low-energy effective action as

$$e^{-S_\Lambda(\phi_{\rm L})} \equiv \int \mathcal{D}\phi_{\rm H} e^{-S(\phi_{\rm L},\phi_{\rm H})} \,. \tag{3.56}$$

The effective action can be expanded in a complete set of local operators $\mathcal{O}_{\Delta_i}$ which consist of powers of the low-energy fields and their derivatives

$$S_\Lambda = S_{\rm CFT}(\Lambda, g^*) + \sum_i \int \mathrm{d}^d x\, g^i \mathcal{O}_{\Delta_i} \,. \tag{3.57}$$

The action $S_{\rm CFT}(\Lambda, g^*)$ is the free action around a fixed point of the beta-function (see below). Normally one takes this fixed point to be the trivial one $\{g^* = 0\}$ which implies that free action is just the kinetic part of the low-energy effective action. For non-trivial fixed points, $S_{\rm CFT}$ can describe an interacting conformal field theory.

Simple dimensional analysis gives for the scaling dimensions

$$\begin{aligned} [\mathcal{O}_{\Delta_i}] &= \Delta_i \,, \\ [g^i] &= d - \Delta_i \,. \end{aligned} \tag{3.58}$$

It is important to note that these scaling dimensions are defined relative to the fixed point of the couplings $\{g^*\}$, and that the value of this dimension can be changed by the interactions. One can then introduce dimensionless couplings by defining

$$\lambda^i = g^i \Lambda^{\Delta_i - d} \,. \tag{3.59}$$

Around energy scales E, we have the following order of magnitude for a typical operator

$$\int \mathrm{d}^d x \mathcal{O}_{\Delta_i} \simeq E^{\Delta_i - d} \,. \tag{3.60}$$

This means that the i-th term in the action is of the size

$$\lambda^i \left(\frac{E}{\Lambda}\right)^{\Delta_i - d} \,. \tag{3.61}$$

The sign of the exponent will determine the relevance of an operator at a given energy scale E compared to the natural energy scale Λ, as we have indicated in table 3.3.2. If the exponent is negative, then for energies much smaller than Λ, the term in the action will become large, and the operator is called relevant. For positive exponents, the term in the action will vanish at low energies – the operator is irrelevant for the low-energy theory. The case of vanishing exponent corresponds to a marginal operator.

A familiar example of theories with an energy scale Λ appears in the orthodox renormalization of quantum field theory. There, Λ is introduced as a regulator, or cut-off, to calculate a

$\Delta_i - d$	Size as $E \to 0$	Type	Theory
< 0	Grows	Relevant	Super-renormalizable
$= 0$	Constant	Marginal	Strictly renormalizable
> 0	Decays	Irrelevant	Non-renormalizable

Table 3.2: Classification of operators in effective field theory.

divergent path-integral. After obtaining a finite answer and renormalizing certain quantities, the cut-off is send to infinity. This is precisely the opposite limit considered in effective field theories.

It therefore follows that irrelevant operators correspond to non-renormalizable theories since they yield infinite terms at high energies. One can nevertheless still make sense of non-renormalizable theories, such as General Relativity, by considering them as low-energy effective theories and only using them at energies far below the cut-off Λ. The dependence of a low-energy effective theory on the high energy physics is only through the marginal and relevant operators.

The scaling derived from simple power counting is modified by interactions in the effective theory; these effects are controlled by the beta-functions. They are defined as follows

$$\beta^i(g) \equiv E\frac{\partial g^i(E)}{\partial E} \,. \tag{3.62}$$

The beta-functions can be calculated in perturbation theory around the fixed point $\{g^*\}$

$$\beta^i(g) = \eta^i g^i + C_{jk}{}^i g^j g^k + \dots \,. \tag{3.63}$$

The constants η^i are called the anomalous scaling dimensions, they measure the deviation from the canonical scaling dimension derived from the free action. The coefficients $C_{jk}{}^i$ appear in operator product expansion of local operators $\mathcal{O}_{\Delta_i}$

$$\big\langle \mathcal{O}_{\Delta_i}(x_i)\mathcal{O}_{\Delta_j}(x_j)\big\rangle_{\text{CFT}} = C_{ij}{}^k(x_i - x_j)\,\langle \mathcal{O}_{\Delta_k}\rangle_{\text{CFT}} \,. \tag{3.64}$$

Of particular interest are the points for which the beta-function vanishes; for these values of the coupling constants, the theory is invariant under a change in scale, and such points are therefore called fixed points. The sign of the derivative of the beta-function at the fixed-points determines whether the fixed-point will be reached for increasing or decreasing energy scale.

We have plotted a typical example of a beta-function in figure 3.1. The arrows on the g-axis indicate in which direction the couplings will flow for increasing energy. The fixed-points at which the slope of the graph is negative are called UV-fixed points since the beta-function will drive the couplings to these values for increasing energies. On the other hand, the fixed-point having a positive slope is reached in the IR.

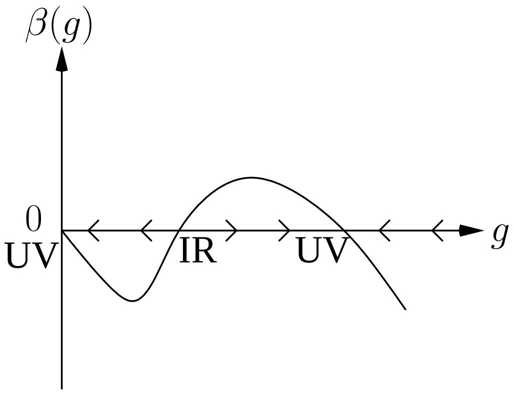

Figure 3.1: A beta-function with UV and IR fixed points.

In coupling space, one can raise and lower indices with the Zamolodchikov metric [128]

$$G_{ij} = |x_i - x_j|^{2d-\eta_i-\eta_j} \left\langle \mathcal{O}_{\Delta_i}(x_i)\mathcal{O}_{\Delta_j}(x_j) \right\rangle_\Lambda \,, \tag{3.65}$$

where the expectation value is now computed with the full effective action S_Λ. The beta-functions are related to a gradient-flow in coupling space [128] also known as a renormalization group flow, or RG-flow

$$\frac{\partial C(g)}{\partial g_i} = -G_{ij}\beta^j(g)\,. \tag{3.66}$$

The C-function is invariant under a change of scale. In particular, in two-dimensions, the C-function is related to the central charge c of the conformal field theory [128], which is proportional to the trace of the energy-momentum tensor

$$C \simeq \langle T^\mu{}_\mu \rangle \,. \tag{3.67}$$

The scale-invariance of $C(g)$ implies that

$$E\frac{\mathrm{d}C(g)}{\mathrm{d}E} = 0 \to E\frac{\partial C(g)}{\partial E} = -E\frac{\partial C(g)}{\partial g_i}\frac{\partial g^i(E)}{\partial E} = G_{ij}\beta^i(g)\beta^j(g) \geq 0\,. \tag{3.68}$$

The last inequality has been proven in two dimensions [128], but no such proof is available in higher-dimensions [129]. The interpretation is that the C-function decreases monotonically form the UV to the IR.

The formalism described above can be applied to the AdS/CFT correspondence in the following way. We saw that, at the boundary of the AdS spacetime at large U, there was a dual description in terms of the UV regime of a conformal field theory. Moreover, fluctuations around the AdS solution corresponded to a correlation function in the conformal field theory of the form

$$\left\langle e^{\int \mathrm{d}^d\vec{x}\,\varphi_0(\vec{x})\mathcal{O}_\Delta(\vec{x})}\right\rangle_{\mathrm{CFT}} . \tag{3.69}$$

This implies that the conformal field theory action is modified with a local operator

$$\int \mathrm{d}^d\vec{x}\,\varphi_0(\vec{x})\mathcal{O}_\Delta(\vec{x}) . \tag{3.70}$$

The coupling $\phi_0(\vec{x})$ correspond to a scalar field $\phi_0(z,\vec{x})$ which has two eigenmodes under rescalings with eigenvalues Δ_+ and Δ_-. The eigenvalue Δ_+ correspond to a relevant perturbation of the conformal field theory inducing a UV-IR flow in the CFT. On the other hand, the Δ_- eigenvalue has the interpretation of deforming the conformal field theory with the vacuum expectation value [130]

$$\left\langle \mathcal{O}_{\Delta_-}\right\rangle_{\mathrm{CFT}} . \tag{3.71}$$

Following the AdS/CFT correspondence, the possible deformations of $\mathcal{N} = 4$ Yang-Mills theory gained new interest [131]. As we will now see, the possible RG-flows that these deformations induce will correspond to interpolating domain-walls in the dual gravity theory [132, 133].

3.3.3 Domain-walls as RG-flows

This section follows to a large extent the treatment of the papers [80, 134]. Recall the toy model from section 2.3.1 of a scalar field with a potential coupled to gravity

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^{d+1}x\sqrt{|g|}\left(R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) . \tag{3.72}$$

We will be particularly interested in potentials of the form

$$V(\phi) = \frac{(d-1)^2}{2}\left(\frac{\partial W}{\partial\phi}\right)^2 - \frac{d(d-1)}{4}W(\phi)^2 . \tag{3.73}$$

The function $W(\phi)$ will be called the superpotential since supergravity theories generically have a scalar potential of the above form. Moreover, it can be shown [135] that potentials of the form (3.73) have stable minima. These minima are related to the following conditions on the superpotential

$$\frac{\partial V}{\partial\phi} = 0 \to \frac{\partial W}{\partial\phi} = 0 , \quad \text{or} \quad \frac{\partial^2 W}{\partial\phi^2} = \frac{d}{2(d-1)}W(\phi) . \tag{3.74}$$

For more realistic models, such as $\mathcal{N} = 8$ gauged supergravity in $D = 5$ which has a potential for no less than 42 scalars, finding the minima of the superpotential is non-trivial.

Nevertheless, for truncations of the total set of scalars, several exact minima have been found for this theory [136].

We saw in chapter 2 that minima of the scalar potential corresponds to Anti-de-Sitter spacetimes. Since gravity in an AdS spacetime should have a holographically dual CFT description, and since deformations of conformal field theories induce RG-flows, we will consider a class of solutions that can be thought of as deformations of Anti-de-Sitter spacetime.

Specifically, we generalize the metric (2.31) to the following form

$$\mathrm{d}s^2 = e^{2A(r)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2\,, \quad \phi = \phi(r)\,. \tag{3.75}$$

In the case $A(r) = -\frac{r}{L}$ we regain Anti-de-Sitter space. The analog of the horospherical coordinates (2.29) is given by

$$\mathrm{d}s^2 = U^2\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \left(\frac{U}{A'(r)}\right)^2 \mathrm{d}U^2\,, \quad U = e^{A(r)}\,. \tag{3.76}$$

For the Ansatz (3.75), the equations of motion (2.53) take on the form

$$\begin{array}{rcl} \phi''(r) + dA'(r)\phi'(r) & = & \frac{\partial V}{\partial \phi}\,, \\ (d-1)A''(r) + \frac{d(d-1)}{2}A'(r)^2 & = & -\frac{1}{4}\phi'(r)^2 - \frac{1}{2}V(\phi)\,, \\ \frac{d(d-1)}{2}A'(r)^2 & = & \frac{1}{4}\phi'(r)^2 - \frac{1}{2}V(\phi)\,. \end{array} \tag{3.77}$$

The equations of motion (3.77) are the Euler-Lagrange equations for the functional

$$E = \int_{-\infty}^{\infty} \mathrm{d}r \frac{e^{dA(r)}}{d-1}\left(-d(d-1)A'(r)^2 + \frac{1}{2}\phi'(r)^2 + V(\phi)\right)\,. \tag{3.78}$$

If the scalar potential is of the form (3.73), then we can use the Bogomol'nyi trick

$$\begin{aligned} E \;=\; & \int_{-\infty}^{\infty} \mathrm{d}r \frac{e^{dA(r)}}{d-1}\left(\frac{1}{2}\left[\phi'(r) \mp (d-1)\frac{\partial W}{\partial \phi}\right]^2 - d(d-1)\left[A'(r) \pm \frac{1}{2}W(\phi)\right]^2\right) \\ & \pm \left[e^{dA(r)}W(\phi)\right]_{-\infty}^{\infty}\,. \end{aligned} \tag{3.79}$$

The extrema of this functional are given by

$$\begin{aligned} \phi'(r) &= \mp(d-1)\frac{\partial W}{\partial \phi}\,, \\ A'(r) &= \pm\frac{1}{2}W(\phi)\,. \end{aligned} \tag{3.80}$$

This means that for scalar potentials of the form (3.73), the second order differential equations (3.77) reduce to a pair gradient flow equations that can be solved by successive quadrature. The equations (3.80) are the same as the ones one would obtain from demanding that the supersymmetry variations of the fermions in the theory vanish. In particular, the supersymmetry variations of the gravitino and the dilatino will take on a schematic form that is similar to (1.75)

$$\begin{aligned} \delta\psi_\mu &= \partial_\mu \epsilon - \tfrac{1}{4}\omega_\mu{}^{ab}\gamma_{ab} + W(\phi)\gamma_\mu\epsilon \,, \\ \delta\lambda &= \partial\!\!\!/\phi - (d-1)\frac{\partial W}{\partial\phi}\epsilon \,. \end{aligned} \tag{3.81}$$

Substituting the spin-connection $\omega_\mu{}^{ab}$ for the metric Ansatz (3.75) into the supersymmetry transformations (3.81), and demanding that these transformations vanish, gives the same pair of first-order equations (3.80) as was derived from the action (2.52). In other words, the flow equations (3.80) actually describe supersymmetric flows.

The scalar field $\phi(r)$ has the dual interpretation of a coupling in the CFT, and from (3.76) we see that $A(r)$ corresponds to the logarithmic energy scale in the field theory. We can then define the analog of the beta-function as

$$\begin{aligned} \beta(\phi) &\equiv U\frac{\partial\phi}{\partial U} \\ &= \frac{\phi'(r)}{A'r} \\ &= -\frac{2(d-1)}{W(\phi)}\frac{\partial W}{\partial\phi} \,. \end{aligned} \tag{3.82}$$

From (3.76), we also deduce that

$$A''(r) = -\frac{1}{2(d-1)}\phi'(r)^2 \,. \tag{3.83}$$

We can then define a C-function [132]

$$C(U) = \frac{C_0}{A'(r)^{2(d-1)}} \,, \tag{3.84}$$

that satisfies monotonicity, something that is not possible to prove from field theory alone [129]

$$\begin{aligned} U\frac{\partial C}{\partial U} &= -2(d-1)C\frac{A''(r)}{A'(r)^2} \\ &= C\left(\frac{\phi'(r)}{A'(r)}\right)^2 \\ &\geq 0 \,. \end{aligned} \tag{3.85}$$

To summarize, the minima of the superpotential, corresponding to AdS spacetimes, are in correspondence with the fixed points of the beta-function. These fixed points are related to

Concept	Domain-wall	RG-flow
Scale	U	E
Log-scale	$A(r)$	$\log E$
Coupling constant	$\phi(r)$	$g(E)$
Beta-function	$\beta(\phi) = \frac{\phi'(r)}{A'r}$	$\beta(g) = E\frac{\partial g(E)}{\partial E}$
Fixed point	AdS spacetime	CFT
C-function	$C \simeq A'(r)^{-2(d-1)}$	$C \simeq \langle T^{\mu}{}_{\mu}\rangle$
C-theorem	$U\frac{\partial C}{\partial U} \geq 0$	only in $d = 2$

Table 3.3: A domain-wall/RG-flow dictionary.

conformal field theories. The induced RG-flow from the UV to the IR between two conformal field theories corresponds in this picture to a domain-wall that interpolates between two AdS spacetimes. We have summarized this domain-wall/RG-flow dictionary in table 3.3.

Using the newly found vacua of $\mathcal{N} = 8$ gauged supergravity in $D = 5$ [136], several interpolating domain-wall solutions were found [137]. These supersymmetric domain-walls correspond to deformations of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory by relevant operators. The induced RG-flows generically have IR fixed-points preserving a smaller amount of supersymmetry, creating hope that also non-supersymmetric gauge theories such as QCD might be described by a dual gravitational theory.